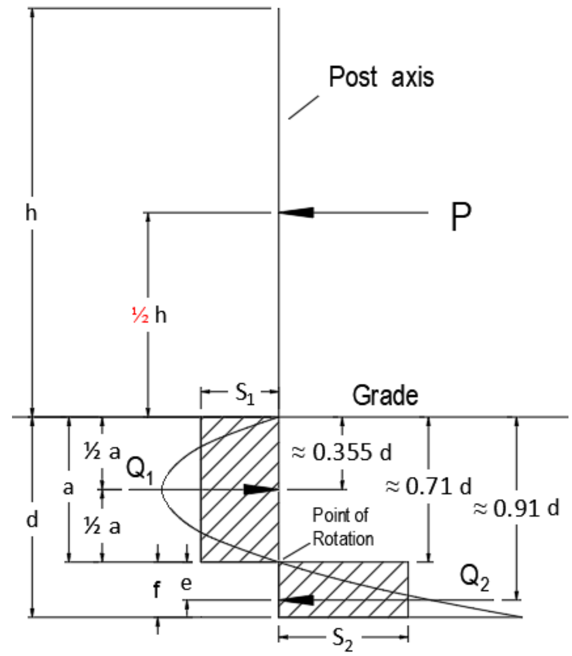
Derivation of the IBC Minimum Depth Formula for Non-constrained Posts



Variable	**Units**
P is the horizontal wind force	lbs
h is the overall fence height	ft
½ h is the force application height for fencing	ft
b is the diameter of the footing	ft
a is the depth of the point of rotation	ft
S_1 is the average soil pressure above the point of rotation	psf
Q_1 is the resultant of the soil pressure above the point of rotation	lbs
S_2 is the average soil pressure below the point of rotation	psf
Q_2 is the resultant of the soil pressure below the point of rotation	lbs
d is the minimum embedment depth	ft
D is the depth being checked D ≥ d is OK	ft

S is the soil Lateral Bearing Pressure $S_1 = \frac{1}{3} D\,S$ $Q_1 = S_1\,a\,b$

Taking the moment about Q_1: $P(\tfrac{1}{2}h + \tfrac{1}{2}a) = Q_2(\tfrac{1}{2}a + e)$

Rearrange for Q_2 formula $Q_2 = \dfrac{P(\frac{1}{2}h + \frac{1}{2}a)}{(\frac{1}{2}a + e)}$

Set Q_1 to sum of opposing forces $Q_1 = P + Q_2$

Replace Q_1 & Q_2 with their formulas $S_1\,a\,b = P + \dfrac{P(\frac{1}{2}h + \frac{1}{2}a)}{(\frac{1}{2}a + e)} = P\dfrac{(\frac{1}{2}a + e)}{(\frac{1}{2}a + e)} + \dfrac{P(\frac{1}{2}h + \frac{1}{2}a)}{(\frac{1}{2}a + e)}$

Condense the formula $S_1\,a\,b = \dfrac{P(\frac{1}{2}h + a + e)}{(\frac{1}{2}a + e)}$ Rearrange the formula $\dfrac{P}{S_1} = \dfrac{a\,b(\frac{1}{2}a + e)}{(a + \frac{1}{2}h + e)}$

Values to match derived formula to IBC $a = 0.6769\,d$ $\frac{1}{2}a = 0.33845\,d$ $e = 0.2408\,d$

$$\frac{P}{S_1} = \frac{0.6769\,d\,b\,(0.33845\,d + 0.2408\,d)}{(0.6769\,d + \frac{1}{2}h + 0.2408\,d)} = \frac{0.3921\,d^2\,b}{(0.9177\,d + \frac{1}{2}h)} = \frac{d^2\,b}{(2.34\,d + 2.55\frac{1}{2}h)}$$

Set up as a quadratic equation ($ax^2 + bx + c = 0$) $(S_1\,b)\,d^2 + (-2.34\,P)\,d + (-2.55\,P\,\tfrac{1}{2}h) = 0$

Calculating the positive solution with the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$d = \frac{-(-2.34\,P) + \sqrt{(-2.34\,P)^2 - 4(S_1\,b)(-2.55\,P\,\frac{1}{2}h)}}{2\,S_1\,b} = \frac{2.34\,P + \sqrt{(2.34\,P)^2 + S_1\,b(10.2)\,P\,\frac{1}{2}h}}{2\,S_1\,b}$$

$$d = \frac{2.34\,P}{2\,S_1\,b} + \sqrt{\frac{(2.34\,P)^2}{(2\,S_1\,b)^2} + \frac{S_1\,b\,(10.2)\,P\,\frac{1}{2}h}{(2\,S_1\,b)^2}} = \frac{2.34\,P}{2\,S_1\,b} + \sqrt{\frac{(2.34\,P)^2}{(2\,S_1\,b)^2} + \frac{S_1\,b\,(4.36)(2.34\,P)\frac{1}{2}h}{(2\,S_1\,b)^2}}$$

$$d = \frac{2.34\,P}{2\,S_1\,b} + \sqrt{\frac{(2.34\,P)^2}{(2\,S_1\,b)^2} + \frac{S_1\,b\,(4.36)(2.34\,P)\,\frac{1}{2}h}{(2\,S_1\,b)^2}\,\frac{(2.34\,P)}{(2.34\,P)}} = \frac{2.34\,P}{2\,S_1\,b} + \sqrt{\frac{(2.34\,P)^2}{(2\,S_1\,b)^2} + \frac{(2.34\,P)^2}{(2\,S_1\,b)^2}\,\frac{S_1\,b}{2.34\,P}\,\frac{(4.36)\,\frac{1}{2}h}{1}}$$

$\boxed{\text{Common Factor, } A = \dfrac{2.34\,P}{S_1\,b}}$

$$d = \frac{A}{2} + \sqrt{\frac{A^2}{2^2} + \frac{A^2}{2^2}\,\frac{1}{A}\,\frac{4.36\,\frac{1}{2}h}{1}}$$

$$\boxed{d = \frac{A}{2}\left(1 + \sqrt{1 + \frac{4.36\,\frac{1}{2}h}{A}}\right)}$$