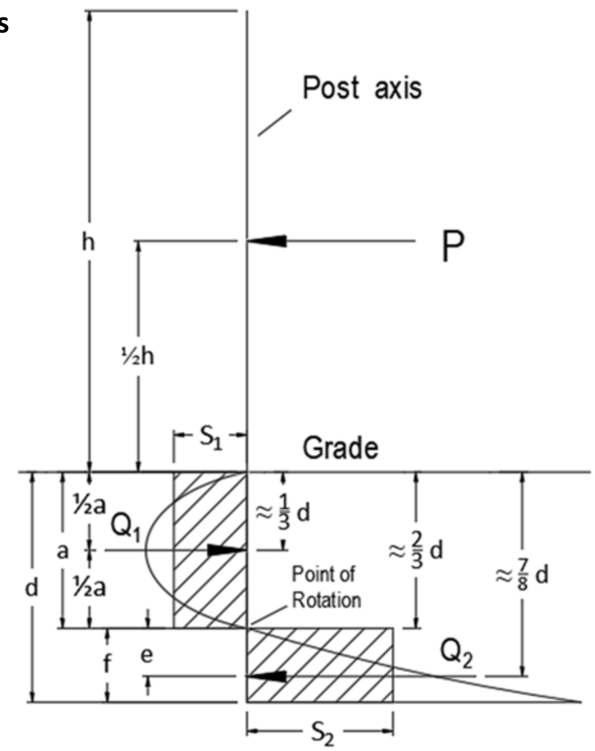
Derivation of the Minimum Depth Formula for Non-constrained Posts

Variable	Units
P is the horizontal wind force	lbs
h is the overall fence height	ft
½h is the force application height for fencing	ft
b is the diameter of the footing	ft
a is the depth of the point of rotation	ft
S_1 is the average soil pressure above the point of rotation	psf
Q_1 is the resultant of the soil pressure above the point of rotation	lbs
S_2 is the average soil pressure below the point of rotation	psf
Q_2 is the resultant of the soil pressure below the point of rotation	lbs
d is the minimum embedment depth	ft



$Q_1 = S_1 a b$

Taking the moment about Q_1: $P(½h + ½a) = Q_2(½a + e)$

Rearrange for Q_2 formula $Q_2 = \frac{P(½h + ½a)}{(½a + e)}$

Set Q_1 to sum of opposing forces $Q_1 = P + Q_2$

Replace Q_1 & Q_2 with their formulas $S_1 a b = P + \frac{P(½h + ½a)}{(½a + e)} = P\frac{(½a + e)}{(½a + e)} + \frac{P(½h + ½a)}{(½a + e)}$

Condense the formula $S_1 a b = \frac{P(½h + a + e)}{(½a + e)}$ Rearrange the formula $\frac{P}{S_1} = \frac{ab(½a + e)}{(a + ½h + e)}$

Values to match derived formula to IBC a = 0.682 d ½a = 0.341 d e = 0.234 d

$$\frac{P}{S_1} = \frac{0.682db(0.341d + 0.234d)}{(0.682d + ½h + 0.234d)} = \frac{0.39215d^2b}{(0.916d + ½h)} = \frac{d^2b}{(2.34d + 2.55½h)}$$

Set up as a quadratic equation ($ax^2 + bx + c = 0$) $(S_1b)d^2 + (-2.34P)d + (-2.55P½h) = 0$

Calculating the positive solution with the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$d = \frac{-(-2.34P) + \sqrt{(-2.34p)^2 - 4(S_1b)(-2.55P½h)}}{2S_1b} = \frac{2.34P + \sqrt{(2.34p)^2 + S_1b(10.2)P½h}}{2S_1b}$$

$$d = \frac{2.34P}{2S_1b} + \sqrt{\frac{(2.34P)^2}{(2S_1b)^2} + \frac{S_1b(10.2)p½h}{(2S_1b)^2}} = \frac{2.34P}{2S_1b} + \sqrt{\frac{(2.34P)^2}{(2S_1b)^2} + \frac{S_1b(4.36)(2.34P)½h}{(2S_1b)^2}}$$

$$d = \frac{2.34P}{2S_1b} + \sqrt{\frac{(2.34P)^2}{(2S_1b)^2} + \frac{S_1b(4.36)(2.34P)½h}{(2S_1b)^2}\frac{(2.34P)}{(2.34P)}} = \frac{2.34P}{2S_1b} + \sqrt{\frac{(2.34P)^2}{(2S_1b)^2} + \frac{(2.34P)^2}{(2S_1b)^2}\frac{S_1b}{2.34P}\frac{(4.36)½h}{1}}$$

Common Factor, $A = \frac{2.34P}{S_1b}$

$$d = \frac{A}{2} + \sqrt{\frac{A^2}{2^2} + \frac{A^2}{2^2}\frac{1}{A}\frac{4.36\ ½h}{1}}$$

$$d = \frac{A}{2}\left(1 + \sqrt{1 + \frac{4.36\ ½h}{A}}\right)$$